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MATHEMATICS — SQUARES, CUBES & RATIONAL NUMBERS

Class: VIII	Max. Marks: 50	Duration: 2 hrs	Date: _____

General Instructions

(i) All questions are compulsory.

(ii) This paper contains 25 questions divided into five sections.

(iii) Section A: 10 MCQs (1 mark each) | Section B: 5 questions (2 marks each) | Section C: 4 questions (3 marks each) | Section D: 3 questions (4 marks each) | Section E: 3 Case-Based questions (3 marks each).

(iv) No overall choice. Internal choices are provided wherever mentioned.

(v) Use of calculators is NOT permitted.

SECTION – A

Questions 1 to 10 carry 1 mark each. (Choose the most appropriate option.)

1. Which of the following CANNOT be a perfect square? *[1 mark]*

(a) 4096
(b) 3025
(c) 2048
(d) 1764

2. If the square root of a number N equals its units digit, which of the following could be N? *[1 mark]*

(a) 25
(b) 36
(c) 49
(d) Both (a) and (c)

3. Which of the following is the standard form of −72/−108? *[1 mark]*

(a) −2/3
(b) 2/−3
(c) 2/3
(d) −72/−108 is already in standard form

4. Which of the following is a perfect cube but NOT a perfect square? *[1 mark]*

(a) 64
(b) 729
(c) 2744
(d) 512

5. If p/q and r/s are two rational numbers such that p/q > 0 and r/s < 0, then p/q + r/s is: *[1 mark]*

(a) Always positive
(b) Always negative
(c) Always zero
(d) Cannot be determined without actual values

6. Using the identity $(n+1)^2 = n^2 + (2n+1)$, what is the value of 44^2? *[1 mark]*

(a) 1936 − 87
(b) 1849 + 87
(c) 1849 + 89
(d) 1936 + 89

7. The units digit of the cube of 47 is: *[1 mark]*

(a) 7
(b) 3
(c) 9
(d) 1

8. Which rational number lies between −3/4 and −2/3? *[1 mark]*

(a) −5/7
(b) −7/12
(c) −11/15
(d) −3/5

9. Assertion (A): The number 1000 is a perfect cube but not a perfect square. Reason (R): For a number to be both a perfect square and perfect cube, all prime factors must appear in groups of 6. *[1 mark]*

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

10. Assertion (A): The rational number −5/−8 is negative. Reason (R): A rational number is negative when numerator and denominator have opposite signs. *[1 mark]*

(a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

SECTION – B

Questions 11 to 15 carry 2 marks each.

11. Find the smallest square number exactly divisible by 8, 9, and 12. *[2 marks]*

12. Find the square root of 3136 by the prime factorisation method. *[2 marks]*

13. Find the cube root of 13824 by prime factorisation. *[2 marks]*

14. What number should be added to −7/15 to get 11/20? *[2 marks]*

15. Find the smallest number by which 1372 must be multiplied to make it a perfect cube. *[2 marks]*

SECTION – C

Questions 16 to 19 carry 3 marks each.

16. Find the smallest number by which 3528 must be divided to obtain a perfect square. Also find the square root of the resulting quotient. *[3 marks]*

17. Arrange the rational numbers −5/6, 7/−9, 11/−18 in descending order. *[3 marks]*

18. Find the smallest number by which 2916 must be divided to get a perfect cube. Also find the cube root of the quotient. *[3 marks]*

19. Simplify: (−5/9 × 27/−35) − (14/15 × −3/7) + (−8/25 × 5/4) *[3 marks]*

SECTION – D

Questions 20 to 22 carry 4 marks each.

20. A sports stadium can seat spectators in a perfect square arrangement. If the current capacity is 7928, find: (a) The minimum number of additional seats needed to make it a perfect square. (b) The number of rows and columns in the new arrangement. *[4 marks]*

21. A rational number when multiplied by 5/7 gives −20/63. What is the rational number? Verify your answer by substitution. *[4 marks]*

22. A cubical container holds exactly 32768 cm^3 of water. A second cubical container has a volume equal to 8 times the first. Find: (a) The side of the first container. (b) The side of the second container. (c) The ratio of their surface areas. *[4 marks]*

SECTION – E (Case Study Based Questions)

Questions 23 to 25 carry 3 marks each.

Case Study 1 – The Chessboard Revisited

Priya read that on a standard 8×8 chessboard, if you place 1 grain on square 1, 2 on square 2, 4 on square 3 and so on (doubling each time), the total grains on the last square equals 2^{63}. She also recalled that perfect squares have an interesting connection to sums of odd numbers. She decided to explore square number patterns on board positions.

(a) Express 64 as a sum of consecutive odd numbers starting from 1.

(b) How many odd numbers are needed to express 225 as a sum of consecutive odd numbers?

(c) Is 200 a perfect square? Justify by attempting the consecutive odd number pattern.

Case Study 2 – Rational Steps

During a hiking trip, Kavya walked 2⅓ km towards north, then 1¾ km towards south, then a further 5/6 km towards north. She wants to track her exact position from the starting point using rational numbers.

(a) Represent each distance as a rational number with appropriate sign (north = positive).

(b) Find her total displacement from the starting point.

(c) If she needs to walk back to the starting point, what distance must she cover, and in which direction?

Case Study 3 – Cube Wonders

Raj's teacher challenged the class with the following: 'The number 1729 is called the Hardy-Ramanujan number because $1729 = 1^3 + 12^3 = 9^3 + 10^3$. Are there other such interesting cube properties?' Raj began exploring perfect cubes and their digit patterns.

(a) Find the units digit of: (i) 34^3 (ii) 75^3 (iii) 99^3

(b) Can a perfect cube end in exactly 3 zeros? Justify.

(c) Verify that $1729 = 9^3 + 10^3$ by actual calculation.